Mathematics Toolkit

DRAFT

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Curriculum Development

Montana Office of Public Instruction
Nancy Keenan, Superintendent

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MATHEMATICS TOOLKIT: CURRICULUM DEVELOPMENT

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INTRODUCTION

Mathematics education has been a matter of national concern for many years. In 1983 *A Nation at Risk: The Imperative For Educational Reform*, a report by the National Commission of Excellence in Education, brought mathematics into the educational spotlight. Since that time there have been many reports published detailing the reforms necessary to provide a quality mathematics education for all students.

The *Curriculum and Evaluation Standards for School Mathematics* (commonly referred to as the *Curriculum Standards*), the *Professional Teaching Standards* (referred to as the *Professional Standards*), and the draft *Assessment Standards*, all published by the National Council of Teachers of Mathematics (NCTM), articulate a vision for progressive change. The current Montana School Accreditation Standards adopted by the Board of Public Education in 1989 reflect the same vision for mathematics education in Montana as the NCTM *Standards* do for the nation.

It is the purpose of this *Toolkit for Mathematics Curriculum Development* to assist schools with the development of local curriculum. By expanding on the vision shared by the NCTM and the Montana mathematics education community, the *Toolkit* more clearly defines a focus necessary to implement curricular reforms. In addition, it contains a curriculum framework that is consistent with the recommendations of the *Curriculum Standards* document and the Montana-based Systemic Initiative for Montana Mathematics and Science (SIMMS) Project, a copy of *Assessment Planning*, written by Jan Hahn and published by the Montana Office of Public Instruction, and a *Guide to Selecting Instructional Materials for Mathematics Education* produced by the Association of State Supervisors of Mathematics and the National Council of Supervisors of Mathematics.

PHILOSOPHY

Educational goals must reflect the needs of society and the needs of students. In this information age of complex technology, our societal needs are changing very rapidly. The use of electronic calculation and communication has caused a dramatic economic shift. "Shopkeeper arithmetic" and a basic working knowledge of the language are no longer sufficient educational aspirations. "New social goals for education include (1) mathematically literate workers, (2) lifelong learning, (3) opportunity for all, and (4) an informed electorate."[1]

Traditional curriculum and teaching practices do not focus on and, therefore, do not meet the new goals for education. Outcomes that specifically address these goals are essential to the reform of mathematics education. Mathematics program outcomes specify the knowledge, process skills, and attitudes that students must learn in order to meet the goals set forth in the shared vision.

Curriculum is only one aspect of teaching and learning mathematics. How the curriculum is taught is as significant as the content that it defines. Recent research recommends that we facilitate the learning process by enabling students to construct concepts using prior knowledge as a base. Students need to explore and discover ideas in ways that encourage internalization and ownership of the learning. When internal construction is the focus of classroom learning, students are empowered to apply knowledge and skills to new situations.

Neither curriculum content nor pedagogy can be separated from assessment. The major purpose of assessment is to cultivate learning by improving both curriculum and pedagogy. Aligning assessment practices with established outcomes will ensure valid assessments that will allow for appropriate modifications to be made. In addition, careful consideration must be given to the specific purpose of each assessment tool used. Many different tools are necessary for meaningful assessment of the development of mathematical power.

Assumptions

Effective teaching practices are necessarily a function of societies' common beliefs about teaching and learning. Educational goals provide a focus for all aspects of instruction and common beliefs dictate the manner in which instruction will take place. The assumptions underlying the development of this toolkit are based on a core of beliefs held by the mathematics education community and evidenced in publications from the Mathematical Association of America (MAA), the National Research Council, and NCTM.

1. *Schools control the conditions for success.* There are many societal and individual problems that schools cannot solve. Various factors

influence school environments. Yet, educators remain the primary influence on school environment.

Schools can control the conditions for success by:

A. *Working in partnership with all stake-holders.* Community members, educators, parents, and students need to be involved in school leadership. Group buy in will establish a supportive environment where all have high standards and expectations.

B. *Empowering teachers.* The empowerment of teachers includes decentralized authority in schools, where teachers are directly involved and held responsible for decision making (Maeroff, 1988). This will require much progress in the move toward "professionalization" of teachers.

C. *Using outcomes that focus on a common vision, to continually assess and improve school programs.* Planning and utilizing a comprehensive program assessment strategy is essential in the progress of an ever changing educational system.

D. *Empowering students.* Students can be empowered by taking appropriate responsibility and by being held accountable for those responsibilities. Students teaching other students can give the mentor student self-esteem, confidence, and interpersonal skills as well as deeper understanding of content materials.

2. *All students can develop mathematical power.* A curriculum based on high-level outcomes will facilitate the learning of conceptual and procedural knowledge of mathematical problem solving, communication, and reasoning.

All students:

A. *Have the potential and desire to be successful.* Varying backgrounds, interests, and learning styles affect the rate at which students achieve. However, a high-level learning focus coupled with high expectations and expanded opportunities will allow all students to reach their potential.

B. *Deserve affirmation.* Appropriate instructional strategies, a diversity of academic and social success experiences, and an

environment that offers security, understanding, acceptance, respect, and direction are affirming for all students.

C. *Are lifelong learners.* People are born with a natural curiosity that can be nurtured into a love of learning, as opposed to an adversity to school, given the appropriate school environment.

D. *Are responsible for their own success.* Teaching students the skills necessary to make informed decisions empowers them to take responsibility for their own learning. Expect students to make choices, trust them to make intelligent decisions, and teach them to be responsible for those decisions.

3. *Teaching is a complex practice that requires constant reflection, evaluation, and modification.* All students, teachers, and school environments are different. There are no set rules that work well for every teacher in every situation. Each teacher must develop teaching practices that best suit their own style and circumstance.

Focus

Project Excellence* resulted in a list of program goals that reflect the vision established by the NCTM in the *Curriculum Standards*. This list of goals, that focus on the established vision, answer the question: What do we want students to know, do and be? Curriculum, instructional, and assessment practices all focus on these program goals.

Each outcome that is a part of the school curriculum directly addresses one or more of the program goals. When developing curriculum, a "design-down" process ensures focus on the established goals.[2] Significant outcomes for the mathematics program, that directly address the goals, are written first. These program outcomes apply to all grade levels. They help to identify outcomes that will enable students to meet the goals.

Enabling outcomes are written that directly address outcomes derived from the program goals. Using this design-down process to create curriculum makes it possible to guarantee that lessons being taught each day address the program goals.

Below is an example of a program goal, several outcomes that address that goal, and some enabling outcomes that will translate into lesson plans.

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* Project Excellence was a statewide effort to develop standards for education in Montana, 1989.

Example Program Goal: *Learn to Communicate Mathematically*

Example Outcome:

Use Oral and Written Language, and Symbols to Communicate Mathematical Ideas

Example Enabling Outcomes:

Write a Paragraph Explaining How the Solution to a Problem Was Reached

Verbally, Explain How Fractions and Decimals are Related to Percents

Example Outcome:

Communicate About Mathematics in Individual, Small Group, and Whole Class Settings

Example Enabling Outcome:

In Small Groups, Discuss Ways in Which Mathematics is Used in Classes Other Than Mathematics and Science

Example Outcome:

Create and Use Multiple Representations to Communicate Mathematical Ideas

Example Enabling Outcome:

Using the Same Data, Develop Different Graphs that Could be Used to Demonstrate Opposing Arguments

Instructional and assessment practices center around program goals, significant outcomes, and enabling outcomes. Assessment and evaluation are based on demonstration of outcomes at all levels. Teaching strategies, classroom environment, and instructional materials are developed to facilitate the learning necessary to enable students to demonstrate outcomes.

When all aspects of the educational environment focus on the shared vision, students will necessarily be *directed toward an opportunity for self-fulfillment*;[3] the ultimate goal of all education.

INSTRUCTIONAL CLIMATE

Administrative Support

Administration plays a key role in the instructional climate of schools. Appropriate resources of time, funds, staff, and facilities are essential to a quality mathematics program. In addition, positive support for developing teaching and assessment materials and good teaching practices is necessary. Local school boards, superintendents, and principals can provide this support by:

1. Ensuring that curriculum and instruction focus on the development of mathematical power for all students.

   The "New Goals for Students" that are listed and explained on page 5 of the Curriculum Standards provide a working definition of the term mathematical power. Mathematical power is the ability to use and apply mathematics and to appreciate its role in diverse areas of our society.

2. Encouraging the empowerment of mathematics teachers.

   A. Provide time and support for involvement from teachers into curriculum, assessment, budget, staffing, and program planning.

   B. Provide for a continuous program of staff development, inservice activities, and evaluation at school district expense.

   C. Ensure that consultant assistance is available to all mathematics teachers from a variety of sources (e.g., conferences, inservice, colleges/universities, opportunity for dialogue, and mathematics specialists).

   D. Encourage constructive interaction among staff members, professional organizations, and community.

3. Budgeting realistically and adequately for the mathematics program at all levels (e.g., manipulatives, resources, technology, release time, travel expenses, and training).

4. Providing informed guidance and counseling for education and career planning in mathematics and related fields.

5. Providing appropriate teacher assessment, including feedback directly to teachers.

School-Community Relations

Reforms necessary for the development of mathematical power for all students require public support. Beliefs and expectations of parents, communities, business, and industry are essential components in working toward this vision. Schools can contribute to educational reform by taking responsibility for school-community relations. These responsibilities may include:

1. Actively involving community members in the school mathematics program.

   A. Establish an advisory committee consisting of local mathematicians, scientists, community leaders, business persons, and parents.

   B. Draw on local resources (such as businessmen, scientists, and applied mathematicians) to enrich classroom instruction and provide additional educational experiences.

   C. Encourage volunteers to become classroom aids, chaperone field trips, and assist with and sponsor local programs to promote and advocate mathematics education.

2. Supporting teacher and student involvement in community activities.

   A. Use student and teacher prepared articles, presentations, and displays to inform the public of current issues relevant to mathematics education.

   B. Volunteer time to community projects.

   C. Open classrooms to observers, student teachers, college/university staff, and inservice training for business and industry.

3. Distributing information about mathematics, mathematics education, and the changing role of mathematics and technology in society, on a regular basis.

   A. Include a mathematics column in the school newspaper, provide information and resources to local media, and report to local civic groups.

   B. Hold public meetings to facilitate discussion of changes that are occurring in mathematics education.

   C. Hold parent/teacher conferences (even in high schools) and open houses. Encourage teachers and parents to communicate throughout the school year, not only at schoolwide conferences.

D. Include local colleges and universities in classroom activities, discussions, decision making processes, and information dissemination.

Learning Environment

A positive learning environment is a function of many conditions that affect student achievement of outcomes and goals. Physical factors such as room size and arrangement, lighting, and coloring are important considerations. Adequate facilities for labs, projects, and group learning are necessary in today's mathematics classroom.

"The teacher is critical in achieving a positive learning environment."[4] That environment must include comfortable interpersonal relationships that are based on mutual respect and consideration. Communication is relaxed and courteous with a focus on making sense of mathematics. Students must be free to question in an environment that encourages critical thinking.

The NCTM *Professional Standards for Teaching Mathematics* (1991) detail how the teacher can establish an environment that fosters the development of each student's mathematical power. The following is adapted from the *Professional Standards*.

To foster an appropriate environment mathematics teachers must:

1. Pose tasks that stimulate students to develop a coherent framework for mathematical ideas.

   A. Focus on problem formulation, conjecturing, problem solving, and mathematical reasoning.

   B. Display sensitivity to, and draw upon, students' diverse background experiences to promote the development of **all** students' dispositions to do mathematics.

   C. Consider mathematical content, students' prior knowledge, learning styles, and process skills such as speculation and investigation.

2. Orchestrate discourse so that each student's thinking is challenged through oral, written, and visual communication.

   A. Ask students to clarify and justify ideas and listen carefully to those ideas.

   B. Provide guidance and leadership during discussions and activities, and encourage each student to participate.

C. Attach mathematical notation and language to students' ideas.

3. Establish norms of discourse so that students listen to, respond to, and question the teacher and each other.

   A. Ask students to make connections, solve problems, and communicate.

   B. Pose tasks that require students to initiate problems, make and investigate conjectures, present solutions, and use mathematical evidence and argument to determine validity of solutions.

   C. Make use of appropriate tools for enhancing discourse (e.g., technology, models, visual representations, terms, symbols, metaphors, analogies, stories, written and oral presentations).

4. Consistently expect and encourage students to make sense of mathematics, display a sense of mathematical competence, and take intellectual risks.

   A. Respect and value students' ideas, ways of thinking, and mathematical dispositions.

   B. Encourage the development of skill and proficiency by providing appropriate contexts, utilizing time, physical space, and materials in ways that facilitate students' learning.

DEVELOPING CURRICULUM

Along with the never-ending process of establishing state standards for accreditation, comes a cycle of local curriculum development. Locally developed curriculum is an important component in the overall philosophy of education in the state of Montana. This process-based philosophy allows schools the necessary flexibility to develop and implement programs that specifically address the needs of communities in Montana.

Effective program development requires strong leadership from professional educators. Although it is the responsibility of the school district to develop appropriate mathematics curriculum, administrators, teachers, board members, parents, students, and community members all need to be involved with and buy into both the process and the program that is being developed. Educational leaders must facilitate communication necessary to ensure community support for the program.

Curriculum development is a monumental task requiring time, expertise, resources, and careful planning for success. This section of the *Toolkit* provides access to resources and furnishes a guideline for planning such a project. Teachers and administrators provide the experience and expertise needed to accomplish program development.

A process for local development of mathematics curriculum is outlined in the following chart and detailed in the following pages. It is suggested that you modify this process by keeping what is useful, discarding what is not, and sequencing steps in an appropriate order for your district.

Mathematics Curriculum
Development Process

Task to be Completed	Target Date	Completion Date
Mathematics Program Improvement Team		
Philosophy		
Assess Current Program		
Program Goals		
Curriculum Development (Materials Selection)		
Implementation		
Continuing Program Assessment		

Curriculum Committee

Forming the mathematics program improvement team (i.e., curriculum committee) is a critical first step in this process. The make-up of the improvement team should guarantee that each stake-holder group is represented and the committee functions efficiently.

Classroom teachers, department heads, administrators, curriculum specialists, school boards, parents, students, business and community members each need to be represented. There are many factors to consider while selecting individuals to represent each group. Choose representatives who are open-minded problem solvers and can work well together. Consider how active and dependable each group member will be.

The role of this committee should be clearly defined. Will it be to write learner outcomes and to match these outcomes with instructional units? Or will the responsibilities stretch beyond this? What outside groups will efforts of this committee need to be coordinated with? Will the committee be responsible for making decisions or will it simply make recommendations to the administration and/or the school board? Will the team be sub-divided into smaller groups? If so, how? Will groups be formed by grade level, by task (i.e., writers, reviewers, consultants) or by school?

Once the role of the committee has been clarified, responsibilities can be identified. It is important to consider both the task and committee members so that all responsibilities can be met. Can the committee attain a realistic understanding of the proposed task, devote adequate time and energy to the project, and become knowledgeable about current research? Are the

necessary resources available to the committee (e.g., *Montana School Accreditation Manual*; support staff or volunteer help for compiling information, typing and duplicating; model curriculums from outside sources; the *Curriculum Process Guide* from the Montana Office of Public Instruction; and the NCTM *Standards*)?

Philosophy

Each district, school, and math department should have a written philosophy statement that articulates beliefs shared by educators, parents, students, and community members. There are many resources available that can help to clarify the vision set forth in these philosophy statements.

Both the Montana School Accreditation Standards for mathematics and the NCTM *Standards* documents define a vision for mathematics education. In addition, the Mathematical Sciences Education Board, the National Research Council, and the Mathematical Association of America have published several reports that address reforms in mathematics education. (Please see resources section for annotated bibliography.) District and department philosophy should be compared to these national reform documents, state reform projects, and current practices. After making comparisons, the committee will be equipped to rewrite, revise, or preserve existing philosophy statements.

Assess Current Program

With a philosophy established, the committee can begin to assess current practices. The main objective is to determine if the mathematics instructional program indeed reflects that educational philosophy. (Please see the Program Assessment section of this *Toolkit* for guidelines to assessing the current program. Other resources for reviewing school mathematics programs are listed in the resources section of this *Toolkit*.)

Program Goals

Program goals are written to facilitate implementation of the philosophy. They provide focus for the entire mathematics program by directly tying the philosophy statement to curriculum and instructional practices. These goals serve as the standards by which all aspects of the program can be judged.

The goals in the Montana School Accreditation Standards (rule 10.55.1401) are listed below.

Mathematics Program
In a basic mathematics program, students:

(a) become mathematical problem solvers;
(b) learn to communicate mathematically;
(c) learn to reason mathematically;

(d) learn to value mathematics;
(e) become confident in their ability to do mathematics; and
(f) select and use appropriate technology to solve problems and acquire new knowledge.

When writing program goals, it may be useful to identify the skills that are most valuable to students. Answer the questions: What do we want kids to know, do, and be? What implications are revealed for the mathematics program? Are all of these implications addressed by the program goals in the accreditation standards?

Choose a starting place for writing program goals. You may choose to modify current goals, start with the six goals listed in the accreditation standards, use an activity designed to brainstorm and clarify goals, or utilize goals written by another district or state. Always compare the Montana Accreditation Standards for the Mathematics Program to your program goals to guarantee that your program will satisfy accreditation requirements.

Curriculum Development

Curriculum is a framework of learner outcomes that bridge program goals and classroom instruction. Units, lessons, and activities address these learner outcomes to enable students to reach the program goals derived from the philosophy. Along with development of student assessment, material selection goes hand in hand with curriculum development. Please refer to the *Guide to Selecting Instructional Materials for Mathematics Education* that is included in the *Mathematics Toolkit.*

The first step in the actual development of curriculum includes formulating the required tasks. Write a work agenda for the committee and include which committee members will be responsible for each task on the agenda and a reasonable timetable. This agenda will constantly be modified as problems and issues develop.

Based on philosophy statements, assessment of the current program, and program goals, choose a model from which to work. A curriculum framework, developed by the Montana Council of Teachers of Mathematics (MCTM), is included with this *Toolkit.* A model from another source such as a school district, a curriculum cooperative, or a state framework may be used. The current district curriculum document is also an option to consider. Although one model will usually be used for the basic format of the new curriculum, ideas from several documents may be incorporated.

Use all available resources to develop a curriculum that is aligned with district and program philosophy. Use the program goals to ensure alignment with district philosophy by writing outcomes that directly address those program goals. Since program goals were also aligned with the mathematics program in the Montana Accreditation Standards, the curriculum will meet accreditation requirements as well.

To ensure alignment with philosophy and program goals and to encourage effective implementation, it is important to ask for input and comments from all who might be interested. Especially encourage all mathematics teachers to review the project in process. Ask for written comments, additions, deletions, and changes. Utilize representatives from various groups to elicit input from other members of the groups that they represent. Analyzing and synthesizing this data may be an extensive task. To make it more manageable, use organizational tools like a summary matrix, ask reviewers to become familiar with the adopted philosophy and goals before commenting on the curriculum, and ask them to summarize their comments.

Throughout the entire process, it is important to persist with the idea that locally developed curriculum is to allow individual districts to meet their unique needs while maintaining the high standards set by Project Excellence and the Montana Accreditation Standards. Efforts to align curriculum with state and national standards must be integrated with efforts to adapt the curriculum to local district needs.

Implementation

To formulate a plan for implementing a new curriculum, decide how administrators, board members, community members, teachers, students, and parents will be involved. Determine what staff development and instructional materials will be necessary. (Please see Instructional Practices and Materials Selection sections of this document.) In addition, determine how much time, money, and other resources will be needed.

Involving all stake-holders in the implementation of a program improvement plan is very important. Initial buy in can be established by including representatives from each group on the development committee; however, that may not be sufficient. Parents and community members can be valuable resources to a school mathematics program when they are given the opportunity to become actively involved. It is essential to make a direct effort to include the entire community in the implementation plan.

Professional development for staff members is essential for progressive change to occur. Any new curriculum aligned with the Montana Accreditation Standards and the NCTM Standards requires active student participation in the learning process. This participation dictates that many mathematics teachers must begin to use alternative teaching styles. Even those who are innovative leaders need continual professional development in order to keep up with the rapid changes occurring in the field.

With the dramatic shifts in the focus of the mathematics curriculum, many teachers and administrators feel ill-equipped to make necessary changes. There is some anecdotal evidence that suggests many teachers lack confidence in their own mathematical backgrounds. Many do not have teaching strategies in their repertoire that lend themselves to active participation by students or that address the dramatic advances in assessment practices and increased attention to communication and reasoning skills. Teachers can be empowered to make informed decisions

as to professional development that will enable them to implement the new curriculum by being involved in the curriculum-development process.

Staff development and community involvement alone will not necessarily produce successful implementation. The availability of appropriate instructional materials is essential. "In the overwhelming majority of classrooms, the content of the textbook determines what is taught and how it is taught."[5] Choosing a textbook that reflects the curriculum is very important; however, it is unlikely that any single textbook will cover the entire curriculum. Additional resources are required in order to adequately reflect the locally developed curriculum. In addition, appropriate teaching strategies, problem situations, and student activities must be part of the program. During the change process, teachers will need support in determining what technology is appropriate for each content area and grade level.

Additional instructional materials necessary for implementation of the new curriculum may not currently be available to teachers. Supplemental teaching materials may be indispensable and computers, software, calculators, and manipulatives are most certainly essential. A plan to purchase the necessary equipment and materials should be supported by both the staff and the administration.

The implementation process will be an extensive one. Carefully thought-out plans that include necessary resources, a reasonable time line, and appropriate involvement of representatives from various groups are keys to success. Continually refer to and revise the plan as appropriate during the implementation process.

Program Assessment

Assessment is an ongoing process in any program and should be done on a continuous basis. Program assessment must be tailor-made for each program in order to assess progress toward unique program goals. In most instances, assessment of the mathematics program will result in formative evaluation. The data gathered will be used to guide decision making in regard to program improvement.

Program outcomes determine the basis for assessment. Multiple forms of assessment are used to determine whether program outcomes are being demonstrated. After data is gathered and organized, it must be analyzed and evaluated. The results of this evaluation process guide program decision making. Program revision and improvement decisions are based on the results. (Please see the Program Assessment section of this document.)

After program assessment and evaluation are complete and recommendations have been made, the process is renewed. Curriculum and instructional practices are reviewed and improved; these revisions are implemented, and once again the program is assessed. Although program assessment, in general, is a summative evaluation process, the process is a never-ending cycle.

INSTRUCTIONAL PRACTICES

Today's mathematics classroom models a laboratory setting. Students do research and experiments; they solve problems using advanced technologies, and they cooperate and communicate with peers during the problem-solving process. Mathematics is learned in contexts with relationships to other disciplines. Students are actively involved in doing mathematics. With this shift in emphasis toward a problem-solving based curriculum must come improved instructional practices.

Mathematics teachers need to include a variety of teaching strategies in their repertoire. Although the traditional lecture style of teaching is still appropriate in some instances, it will not suffice as a sole teaching strategy. Use of technology, learning styles, connections to other disciplines and application of concepts, along with past experience of students are all factors that must be considered while developing and using various strategies.

Cooperative Learning

Cooperative learning strategies provide a setting in which students can become actively involved in the learning of mathematics. As described in the *Professional Standards for Teaching Mathematics* (NCTM, 1991), student discourse is an important component of this active learning. Discussion among peers facilitates the construction of new ideas. Students can ask and answer questions, make conjectures, formulate and articulate convincing arguments, and present their own ideas. Cooperative learning groups help students to become less dependent upon the teacher. They are compelled to rely on each other, their own reasoning skills, and mathematical evidence to determine the validity of arguments and the reasonableness of solutions.

Group Process

Cooperative learning involves much more than moving desks together or simply allowing students to work in groups. Meaningful discussion does not happen just because students are given a good problem and allowed to talk about it. Teachers cannot expect students to solve problems cooperatively without being taught the necessary skills.

Group skills include communication, leadership, and cooperation. Effective interaction requires practice with speaking and listening skills and experience in differing group roles. Students must know and understand the benefits of group work. Since students will be expected to solve mathematics problems using a group process, it makes sense to first teach students group skills and allow them to practice these group skills before they are asked to use them in problem-solving situations. (Please see the section of this *Toolkit* entitled "Preparing Students to Learn" for an example lesson plan to teach the group process.)

Group Guidelines

Since many students learn early in their academic careers that talking to other students and questioning the teacher are inappropriate behaviors, implementing group activities can be very difficult. On the other hand, many students may see group situations as perfect opportunities to discuss personal matters or otherwise disrupt class. Group guidelines are designed to help the teacher manage classroom discourse by providing a clear definition of expected behaviors.

These guidelines can be provided by the teacher, developed by students, or be derived through some combination of teacher and student input. It is essential, however, that both the teacher and the students agree that guidelines are fair and valid. In addition, each must have a clear, shared understanding of the meaning of each guideline. (See "Preparing Students to Learn" for example guidelines.)

Strategies

Like group process skills, strategies for organizing group work must also be taught to students. Different situations are more effectively managed with various organizational structures. Brainstorming may be the best strategy for the generation of new ideas while small group discussion may be best suited for the development of already formulated ideas. Practicing various strategies will give students a wide range of experiences on which to build problem-solving skills. (The "Preparing Students to Learn" section of this document includes various group strategies and examples of how to teach them.)

Group Skills

The group process learner goals are included in this section because they are fundamentally different from the outcomes described in the curriculum framework. These group skills are important for mathematical problem solving, but are transferable to all group situations.

Once again, it is important for students to practice these skills in many different situations. Teaching group skills prior to and in conjunction with mathematics lessons will strengthen both problem-solving skills and group-process skills. These outcomes may serve as a guideline while developing teaching strategies that incorporate cooperative learning strategies.

Group Process Learner Goals

Demonstrate effective group interaction
(e.g., in pairs, small groups, and whole class situations).

Establish positive interdependence.

- Define group goals.
- Articulate group goals.
- Reach group consensus.

Develop interpersonal communication skills.

- Model listening skills.
- Employ questioning techniques.
- Incorporate constructive criticism.
- Utilize peer tutoring, peer and self-assessment.

Peer and Self-Assessment

Peer and self-assessment practices have a number of instructional advantages. As with all assessment practices, the major concern is to improve student learning.

Teaching students to assess their own work provides an atmosphere in which students are encouraged to value the quality of that work. By establishing clear criteria and providing opportunities to practice self-assessment techniques, teachers can enable students to produce high quality work with confidence.

Peer assessment practices provide students with opportunities to practice group skills, discuss mathematics and problem solving, experience differing approaches to similar problems, and reflect on their own mathematics, communication, and reasoning skills. Effective peer assessment techniques require a great deal of effort from teachers and students alike. The following tips may help you to implement practical and productive peer assessment practices.

1. *Provide students with rich opportunities to practice peer assessment techniques.* Do not expect students to fairly assess work done by other students before they have had opportunities to practice the necessary skills. Being able to identify incomplete work, misrepresented ideas, errors, strengths, and exceptional work is only one of the necessary skills. The ability to discuss these findings in a positive, cooperative, and helpful manner is also essential. Presenting and accepting constructive criticism, encouraging advice, and authentic compliments requires much practice.

2. *Teach students to relate assessment to criteria.* Criteria for assessment must be established before work begins. All students involved and the teacher must have a clear understanding of the assessment criteria. When students are practicing peer assessment ask them to relate each comment, whether it is a criticism or a compliment, to the established criteria.

3. *Hold students responsible for their assessments of other students' work.* Make it clear to all students the peer assessment process has benefits for all involved and that each student will be held accountable for his/her responsibilities.

With the introduction of authentic instructional and assessment practices into the mathematics classroom, teachers can become overwhelmed. Effective self and peer assessment practices can serve to relieve the teacher of some of the long hours pouring over poorly done papers, projects, and presentations. As students become more confident in themselves and each other, learn to rely on their own mathematical knowledge and reasoning skills, and become proficient in self and peer assessment techniques, the quality of work produced will increase. (The "Preparing Students to Learn" section includes an example for teaching peer assessment techniques.)

Peer Tutoring

Incorporating peer tutoring into the mathematics classroom offers several advantages. Like peer and self-assessment, peer tutoring can help the teacher to handle the many duties and responsibilities that tend to become unmanageable, especially during a period of transition. More significant, however, are the direct benefits to students. All students will encounter periods during their academic career when some individual tutoring would be most beneficial. Often times explanations from a peer tend to be more relevant and more easily understood. At the same time, the student who is doing the teaching gains valuable experience. The skills learned from one-on-one interaction have obvious merit. In addition, the tutor is given the opportunity to view the content from a different perspective, present an explanation of the concept, and be exposed to another students' thinking processes.

Peer tutoring can be used in a very formal setting or within the group process. Skills similar to those used in peer assessment are essential in all peer tutoring situations. In order to teach students the skills necessary to tutor one another effectively, it may be beneficial to begin with a very structured process. (There is an example of a structured peer tutoring process in the "Preparing Students to Learn" section of this document.)

ASSESSMENT

The *Montana School Accreditation Standards and Procedures Manual* requires each school district "..in all program areas and at all levels...(to) assess student progress toward achieving learner goals...". In addition, these standards require student assessment to "...be used to examine the program and ensure its effectiveness." This necessarily dictates that each district must develop a program assessment in which the assessment of learner goals is used as a basis for improving the mathematics program.

Assessment Planning

Assessment practices must be carefully thought out, deliberately planned, and continually re-evaluated. Assessment planning can be carried out in a fashion similar to that of curriculum development. First a committee is formed, the responsibilities of the committee are established, and then duties are assigned. Teachers, administrators, school boards, students, and community members should all be involved in the planning process.

Essential first steps to an assessment plan are writing a statement of philosophy and an assessment policy. The 1993 Montana Office of Public Instruction document *Assessment Planning: A Process Guide with Three Design Options*, written by Language Arts Specialist Jan Hahn, contains example philosophy statements and assessment policies. This excellent resource serves as a guide throughout the planning process and is included in the *Toolkit for Mathematics Curriculum Development.*

PROGRAM ASSESSMENT

Program assessment consists of many components. The top priority may be assessing student progress toward learner goals, as required by the accreditation standards. However, also very important are curriculum, instruction and evaluation practices, quality of personnel, and community involvement and support. Each component of the mathematics program has an impact on student learning; therefore, each should be evaluated in order to improve all aspects of the program.

Since there are multiple aspects of the program to be evaluated, it is essential to carefully choose a reasonable number of goals to assess. Only gather information that may lead to program improvement. Do not waste valuable time and other resources collecting data that cannot or will not be used.

Once it has been decided to evaluate a particular program-related goal, assessment instruments must be developed. Curricular outcomes, behavioral goals, teaching techniques, and assessment practices are constantly changing. These continual changes make it difficult to find valid assessment instruments. That is, instruments that actually address current student learner goals, or that investigate whether or not a program is being delivered as it was intended.

A Guide for Reviewing School Mathematics Programs, published by the National Council of Teachers of Mathematics and the Association for Supervision and Curriculum Development in 1991, includes 58 pages of matrices designed to guide mathematics program assessment. The Montana Office of Public Instruction published *Program Assessment: A Six-Step Process to Curriculum Improvement* in 1993 that devotes a section to suitable assessment instruments. If possible, local school districts should adapt existing instruments to fit local circumstances or develop instruments that are appropriate to their own program.

Program Assessment Instruments

Each of the assessment instruments in this section are merely examples. If used, they should be adapted to fit the needs of the local school district. Each example is specifically designed to be different that the others in order to provide a variety of formats. Each format may be adaptable to any component of the program that is being evaluated.

Program outcomes are taken from the curriculum framework that accompany this document and should also be modified to suit local district philosophies, mission, and goals.

Curriculum Assessment

It may be appropriate to ask an outside evaluator to begin the assessment of your curriculum document. Below are some example questions that an objective evaluator might be asked to consider.

Does the curriculum document

- meet required accreditation standards?
- reflect the district's philosophy as defined in the board-adopted philosophy statement?
- tie directly to the philosophy adopted by the mathematics department?
- indicate that the NCTM *Standards* were considered in its development?

Is the curriculum

- specific enough to be usable?
- so specific that teachers have no flexibility?
- clearly organized?

After an outside evaluation has been done, you may choose to do extensive rewriting of the curriculum, make minor changes, or continue the evaluation process by involving the staff, students, school board and community members. Below is an example form that could be used to record the opinions of mathematics teachers and other in-house evaluators.

Problem Solving K-12	Excellent	Needs Work	Comments
Identify, formulate, and clearly define problems from within and outside mathematics.			
Investigate, research, and explore problems to determine best possible solution(s).			
Present, interpret, justify, and evaluate solutions, predictions, and recommendations.			

Communication K-12	Excellent	Needs Work	Comments
Develop common understandings of mathematical ideas.			
Appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas.			
Reflect upon and clarify mathematical thinking.			

Reasoning K-12	Excellent	Needs Work	Comments
Validate, clarify, and explain mathematical thinking.			
Make and evaluate mathematical conjectures, arguments, and conclusions.			

Assessment of Student Progress

Certainly the most important measure of a quality program is the skills and knowledge that students learn. Valid assessment of learner goals must fit the locally developed philosophy and be directly tied to the curriculum. A K-12 curriculum document may contain hundreds of student outcomes; obviously not all of them can be evaluated at the program level. Choose only the most significant outcomes for program evaluation.

High-level outcomes that require understanding of concepts and development of skills as opposed to memorization of facts or algorithms will be remembered for years after graduation. Program assessment should focus on those concepts and skills that will be important even after the "details" are forgotten.

The following is an example mathematics assessment plan. The alternative assessment techniques listed are described in the student assessment section of this *Toolkit*. An example plan for using portfolios is included in the OPI *Assessment Planning* document.

How is the Evaluation Done?			
	Written Tests and Activities	Performance	Observation/ Conferences
Problem Solving	Present Solutions, Explain Processes, Solve Word Problems	Open-Ended Problems, Extended Problem Solving	Checklist of Problem-Solving Skills
Communi-cation	Journals, Portfolios	Presentations	Checklists, Anecdotal Records, Video Tape Groups
Reasoning	Journals, Teacher-Made Tests, Explain Word Problems	Solve Problems, Justify Solutions	Questioning, Checklists, Interviews

Who Does the Evaluation?			
	Student (Self)	Teacher	Community/Other
Problem Solving	Determine Reasonableness of Results, Document the Problem-Solving Process	Checklist of Skills, Open-Ended Problems, Extended Problems	Real-Life Problems, Projects (e.g. Science Fair) On-the-Job
Communi-cation	Goal Setting, Self-Reflection, Journal Writing	Presentations, Journals, Observation	Presentations, Group Projects, Open-House, Portfolio Parties
Reasoning	Explain Reasoning to Peers, Self-Questioning	Teacher-Made Tests, Explanation of Process, Word Problems	Mathematics Contests, Decision Making

Assessment of Instruction and Evaluation

Assessment and evaluation should reflect the district philosophy and the philosophy adopted by the mathematics department. In addition, they should be directly tied to the curriculum. Instruction must direct student learning toward the outcomes prescribed in the curriculum. Evaluation should be designed to assess student progress toward those same outcomes.

The following is an example instrument that could be used to encourage teachers to engage in reflective thinking about their own instructional and evaluation practices.

Teacher Questionnaire

1. Explain how you ensure that all students have equal opportunity to learn, given varied learning styles.

2. What do you do to promote open communication in the classroom? (Give an example.)

3. Give several examples that demonstrate effective use of cooperative learning in your classroom.

4. Explain how you hold students accountable for self and peer evaluation skills.

Personnel

Personnel may be the most difficult component of the mathematics program to evaluate. Although teacher evaluations done by administrators are common practice, it is less frequent that teachers do self and peer evaluations. The following survey is designed to help teachers evaluate their own teaching practices.

Teacher Survey

Rate the following questions on a scale from 1 to 10.
(1=Never, 5=Sometimes, 10=All of the Time)

To foster an appropriate environment you:

1. Pose tasks that stimulate students to develop a coherent framework for mathematical ideas.

_____ A. Focus on problem formulation, problem solving, and mathematical reasoning.

_____ B. Display sensitivity to, and draw upon, students' diverse background experiences to promote the development of all students' dispositions to do mathematics.

_____ C. Consider mathematical content, students' prior knowledge, learning styles, and process skills such as speculation and investigation.

2. Orchestrate discourse so that each student's thinking is challenged through oral, written, and visual communication.

_____ A. Ask students to clarify and justify ideas and listen carefully to those ideas.

_____ B. Provide guidance and leadership during discussions and activities, and encourage each student to participate.

_____ C. Attach mathematical notation and language to students' ideas.

3. Establish norms of discourse so that students listen to, respond to, and question the teacher and each other.

_____ A. Ask students to make connections, solve problems, and communicate.

_____ B. Pose tasks that require students to initiate problems, make and investigate conjectures, present solutions, and use mathematical evidence and argument to determine validity of solutions.

_____ C. Make use of appropriate tools for enhancing discourse, (e.g., technology, models, visual representations, terms, symbols, metaphors, analogies, stories, written and oral presentations.)

4. Consistently expect and encourage students to make sense of mathematics, display a sense of mathematical competence, and take intellectual risks.

_____ A. Respect and value students' ideas, ways of thinking, and mathematical dispositions.

_____ B. Encourage the development of skill and proficiency by providing appropriate contexts, utilizing time, physical space, and materials in ways that facilitate students' learning.

Administrators also play a significant role in providing a quality mathematics program. School boards, superintendents, and principals may find the following survey useful in a self-evaluation process.

Administrator Survey

Rate the following questions on a scale from 1 to 10.
(1=Never, 5=Sometimes, 10=All of the Time)

You provide positive support by:

_____ 1. Ensuring that curriculum and instruction focus on the development of mathematical power for all students.

_____ 2. Encouraging the empowerment of mathematics teachers.

- _____ A. Provide time and support for involvement from teachers into curriculum, assessment, budget, staffing, and program planning.
- _____ B. Provide for a continuous program of staff development, inservice activities, and evaluation at school district expense.
- _____ C. Ensure that consultant assistance is available to all mathematics teachers from a variety of sources (e.g., conferences, inservice, opportunity for dialogue, and mathematics specialists).
- _____ D. Encourage constructive interaction among staff members, professional organizations, and community.

_____ 3. Budgeting realistically and adequately for the mathematics program at all levels (e.g., manipulatives, resources, technology, and training).

_____ 4. Providing informed guidance and counseling for education and career planning in mathematics and related fields.

Community Involvement and Support

Positive school-community relations can be an important component to a successful education program. When schools accept the responsibility to promote these community ties, the program is strengthened. The following questions can help a school to determine how actively community relations are being encouraged.

Check All That Apply

In an effort to promote school-community relations I:

_____ 1. Actively involve community members in the school mathematics program.

_____ A. Establish an advisory committee consisting of local mathematicians, scientists, community leaders, and parents.

_____ B. Draw on local resources (such as businessmen, scientists, and applied mathematicians) to enrich classroom instruction and provide additional educational experiences.

_____ C. Encourage volunteers to become classroom aids, chaperone field trips, and assist with and sponsor local programs to promote and advocate mathematics education.

_____ 2. Support teacher and student involvement in community activities.

_____ A. Use student and teacher prepared articles, presentations, and displays to inform the public of current issues relevant to mathematics education.

_____ B. Volunteer time to community projects.

_____ C. Open classrooms to observers, student teachers, college/university staff, and inservice training for business and industry.

_____ 3. Distribute information about mathematics, mathematics education, and the changing role of mathematics and technology in society, on a regular basis.

_____ A. Include a mathematics column in the school newspaper, provide information and resources to local media, and report to local civic groups.

_____ B. Hold public meetings to facilitate discussion of changes that are occurring in mathematics education.

_____ C. Include local colleges and universities in classroom activities, discussions, decision-making processes, and information dissemination.

Along with information gathered from school staff, assessment of community support must include information gleaned directly from the community. The following is a survey designed to sample the attitudes of the community.

Community Survey

Please check all that apply:

- ☐ Parent of a School-Aged Child
- ☐ Parent of a Grown Child
- ☐ Leader of a Youth Organization (e.g., little league, 4-H, church group,...)
- ☐ Community Member
- ☐ Member of a Senior Citizen's Group
- ☐ Business Person
- ☐ Educator
- ☐ Government Agency
- ☐ Member of a Civic Organization
- ☐ Other _____

What makes you feel welcome and comfortable at _____ School?

_____

_____

Has someone from the school ever contacted you in regard to becoming involved in a school program or function? What made the outcome positive or negative?

_____

_____

What school programs have you been involved in?

_____

_____

Have you ever served on a curriculum or policy committee for the school district?

_____

_____

How can the school help you to become more involved in school related activities?

_____

_____

What one thing do you think that schools could do to improve school-community communication?

_____

_____

What one thing could parent/community members do to become more active in school programs?

_____

_____

Analyzing and Using Data

After the questions have been asked and the surveys have been returned, the data must be analyzed. Data analysis can be done in many ways. The most apparent may be tallying the responses and then reporting what percent of respondents reported a particular response. For example, one in ten parents may report that they have been contacted by the school. Presentations of results should be clear and easily understood.

Use easy to interpret bar and circle graphs, percentages, and totals when presenting data to school boards, other educators or community members. Your presentation of data may well affect the decision that school board and administrators are being asked to make.

In addition to the traditional techniques of quantifying data, you may want to carefully read the antidotal data that is presented. Can it be summarized by an objective reader? Does it support the "numbers"? Are suggestions for improvement present in the comments received?

Regardless of how the data is compiled or presented, it should be used as a basis for improving the mathematics program provided by the school. Decisions can be made based upon program strengths and weaknesses. Some aspects may be working very well while others should be abandoned completely. Those decisions can be made based on the data presented.

Program assessment is a continuing cycle. Constant assessment, evaluation, and program revision are necessary to keep pace with the rapidly changing world.

STUDENT ASSESSMENT

The vision described in the NCTM *Standards* documents is changing the focus of student learning. With an increased emphasis on problem solving, critical thinking, and decision making, current reform efforts are dictating change in curriculum as well as instructional practices. Students utilize technology to solve complex problems. They incorporate high level reasoning and communication skills to make recommendations and predictions based on results of their problem solving.

As the paradigm on which we base our perceptions of education change, so must the assessment practices that we employ in order to evaluate student progress. The focus of assessment must shift according to the transformations that are occurring in curriculum and instruction. Significant outcomes in the mathematics curriculum focus on high level problem-solving, reasoning, and communication skills. It follows that student assessment must focus on the same high level skills.

Appropriate assessments clearly address significant outcomes. Assessment activities must enable students to demonstrate the skills defined by these outcomes. Traditional mathematics assessments address a very small portion of the curriculum described in the NCTM *Standards* and often do not consider methods of instruction or learning styles. Although some traditional assessment instruments are appropriate in various situations, development of alternative, more authentic assessments is essential.

Assumptions

The tools for student assessment that are included in this section are based on the fundamental premise that mathematics education is moving toward a standards-based educational system. Assessments are a means of measuring, observing, and recording educational accomplishments in order to evaluate progress toward goals. Evaluations made on the basis of appropriate assessments are then used, primarily to improve learning. The following assumptions are keys to the development of student assessments.

1. *Evaluations are made based on what students can do; therefore, assessment instruments should focus on what students can do as opposed to what they cannot do.* A holistic approach to assessment that incorporates a problem-solving strategy encourages inventive thought and enables students to apply knowledge and skills. Traditional testing methods are very narrow in scope and do not allow students to demonstrate a wide range of skills nor do they allow for differing learning styles, development of innovative thought, or unexpected but appropriate responses.

2. *Assessment criteria should be well-defined and known to students early in the instruction process.* It is important for students to know and understand the criteria used for assessment and evaluation. Assessment criteria should be used

to define educational goals and students should be involved in determining these criteria. When goals and assessments are known in advance, the results are a measure of what students know and can do as opposed to how well students can anticipate what they will be tested over.

3. *Students should be assessed on all significant outcomes.* Curriculum development, in a nutshell, is the answer to the question: What Should students know, do, and be like? Like curriculum development, assessments need to cover knowledge, skills, attitudes, and behaviors. Assessment should include group skills, communication skills, and confidence in one's abilities to do mathematics.

4. *Instruction and assessment activities should be integrated.* Rich problem-solving activities are excellent sources for instructional tasks that can easily incorporate assessments. Teachers have always made constant informal evaluations during instruction. It is equally valuable to use assessment activities as additional opportunities for instruction.

Changing Assessment Practices

Change can be a long and difficult process. There are many advantages to relying less on traditional forms of assessment. However, using rich problem-solving based situations to enable students to demonstrate understanding of mathematical concepts requires many changes in the educational system. Ultimately, it is teachers who implement these changes. The following ideas may help you through the change process.

1. *Start small.* Do not try to change all of your assessment practices at one time. Choose a simple assessment activity that directly addresses one outcome and give it a try. Add authentic assessments to your repertoire one at a time. Work on improving each new idea as you progress.

2. *Involve all stake-holders in the development of new assessment practices.* Explain the concepts of mathematical power, high-level thinking skills, and authentic assessment to students and parents. Ask for input when designing assessment activities. Involve community members and parents by asking them to present how mathematics is used in their work place and what mathematics skills employers will expect.

3. *Devise an assessment plan.* Develop a timeline for change. Write down what changes that you want to achieve and how you will know when they have occurred. (*Mathematics Assessment* published by the NCTM contains a section on developing an assessment plan. *Assessment Planning: A Process Guide with*

Three Design Options published by the Montana OPI is included in the *Mathematics Toolkit.)*

4. *Gain professional support.* Present alternative assessment ideas and your assessment plan to administrators and to the school board. Work with teachers in your building and district when developing and implementing alternative assessments. Become actively involved in the Montana Council of Teachers of Mathematics. Become a regular user of the METNET Bulletin Board System. Attend local, regional, and national professional meetings.

5. *Carefully document new assessment practices.* Validity of traditional testing has come into question (i.e., Do the tests actually measure what we most want kids to learn)? The same question must be answered of newly developing assessment instruments and practices. Keeping descriptive reports of student progress, observations, oral presentations, group activities, and demonstrations of skills will help to clarify the validity of these measures. In addition, these records can be used as reports to parents and to aid in the improvement of curriculum, instruction, and assessment practices.

Tools for Alternative Assessment

Rubrics

A rubric provides a blueprint of an assessment. It's primary function is to clarify and express the criteria for an intended performance. This tool allows you to combine assessment with instructional activities.

Essentially, there are two kinds of rubrics: analytical and holistic. An **analytical rubric** breaks performance down into its component parts. Level of performance for each aspect of an assessment is determined according to the established criteria. An overall evaluation of the performance is made only after all criteria have been met. A **holistic rubric** considers performance as a whole. The feedback is less specific, but it allows for much faster processing of assessments.

Rubrics can be used with any performance assessment. They provide a way for teachers to inform students of criteria, turn instructional activities into assessment opportunities, and to document student performance. Included below are models that may be used to create several different forms of rubrics.

Analytical Rubric
Example Format

Name _____ Date _____

Peer Assessor _____

Project/Assignment _____

	Criteria	Self		Peer		Teacher	
		Ex	In	Ex	In	Ex	In

Ex = Excellent
In = Incomplete

Holistice Rubric
Example Format

Name _____

Date _____

Peer Assessor _____

Project/Assignment:

Excellent	Details the requirements for an excellent performance.
Competent	Lists and describes the criteria for a competent performance.
Incomplete	Describes a performance that needs additional work. May give examples of items that are missing or depict a performance that has specific criteria missing or that are poorly demonstrated.

Portfolios

Portfolio assessment can be a useful tool when attempting to capture a record of the learning process. It can provide continuous feedback to students, as well as a context that encourages self-reflection and self-evaluation. Careful planning is the key to successful portfolio assessment. Simply creating a folder of student work will not provide all of the advantages inherent in the alternative assessment process.

Planning for development of a portfolio must first consider the purpose of the portfolio. If the portfolio will be used as part of an application for a mathematics scholarship, the student will want to develop a presentation of his best work. In contrast, if the teacher would like to use a portfolio to assess student progress while practicing the problem-solving process, a complete record of all activities, including notes, explanations, and rough drafts, should be included. More commonly, portfolios are used to keep a record of chronological development of student progress. These portfolios may be used for reporting to parents, permanent records for individual students, or holistic assessment of student progress.

With a clear picture of the purpose for developing a portfolio, start to plan. The following questions may help guide you through the planning process:

1. *Who will decide what to include in the portfolio?* Students may be free to choose what goes into their portfolio or the teacher may ask all students to include the same assignments. More appropriate may be a combination of student choices and teacher guidelines. If a purpose of the portfolio is to encourage students to practice self-reflection, it is imperative that students be involved in the selection process.

2. *What are the criteria by which selections and evaluations will be made?* Clearly defined and communicated criteria empower students to take responsibility for their own learning. Establishing criteria for assessment and evaluation is a way to articulate to students what is important for them to learn. Criteria should guide the student and the teacher in making choices concerning items to be included, but more importantly these criteria will allow students to assess their own progress continually.

3. *How will the portfolio be used in assessment?* Portfolio assessment can be used in a variety of ways. Individual student assessment, assessment of group work, and program assessment may all be valid uses of the portfolio. Although the vast majority of teachers are forced to reduce all student work to a final letter grade, to some it seems inappropriate to assign grades to student portfolios. Reducing the activity to a rating scale seems contradictory to the purpose of "authentic assessment."

4. *Will integration with instruction occur?* Critical-thinking skills, decision making, and problem solving are all major components of today's mathematics curriculum. Each are vital skills while producing a portfolio and serve to empower students. Through the practice of self-reflection and evaluation skills, students learn to value their own learning. Improvement of instruction and learning remain the most important aspect of assessment practices and are benefits that can be incorporated into portfolio assessments when integration between instruction and assessment occurs.

5. *What potential problems may be encountered?* As with any assessment instrument, results of portfolio assessments can be misleading, misinterpreted, or used to misrepresent actual learning. It is the responsibility of all parties involved to use portfolios in an appropriate manner. Criteria for selection of materials must be known to all who may view or otherwise judge student portfolios. In addition, the criteria must be aligned with curriculum, instruction, and assessment practices.

Checklists

The checklist is primarily used to facilitate record keeping. However, like the rubric, the checklist can help to clarify expectations. For example, teacher observation may be the primary method of assessing group process skills. A checklist makes it easy for the teacher to record observations. In addition, having prepared a list of desired behaviors serves to identify clear criteria that both the teacher and students can refer to. A checklist is a tool used to make methods of alternative assessment manageable and can be used with a variety of assessment strategies. The following example provides a method for the teacher to record observations of behaviors that demonstrate group process skills.

Group Process Skills
Assessment Example Checklist

STUDENT	Speaking	Listening	Group Role	Questioning

Enter the date that the student was observed demonstrating appropriate behaviors. Include comments when possible.

Methods of Assessment

Open-Ended Tasks

Open-ended tasks present a problem that may or may not have a single correct solution. Students are allowed to approach the problem from a variety of directions and are asked to include a complete explanation of their thinking process. Criteria for assessment should be clearly defined and provided upon assignment of the task.

Open-ended problems can be used in a variety of formats including individual, small group, and whole class situations. These problems provide a convenient occasion to incorporate assessment opportunities into instructional situations. For example, you may pose a problem to be worked on individually as homework. The next day in class, small groups of students can use peer tutoring and peer assessment techniques in order to finalize individual solutions. Final solutions may be presented in writing or orally to the entire class.

Authentic problem solving is an excellent source of open-ended tasks. Problems that may have more than one solution or that can be solved in a variety of ways provide a rich resource for students while they are developing their own repertoire of problem-solving techniques. Written, oral, and visual presentations of why algorithms work help to increase understanding of content as well as increase communication and reasoning skills. Nearly any traditional math problem can be improved by asking students to explain how to solve the problem and why the method works. Below are several examples including criteria for evaluation.

1. *When a natural number is divided by a number between zero and one, the quotient is larger than either the dividend or the divisor. Explain.*

   Criteria: *Use manipulatives to model an example.*
   Correctly use the terms quotient, dividend, and divisor.

2. *Explain why the algorithm for division of fractions is invert and multiply.*

   Criteria: *Explanation is clearly presented.*
   Use a concrete example.
   Incorporate your explanation of question number one.

Extended Problem Solving

Extended problem-solving tasks are similar in nature to open-ended problems. They are often broader in scope and require a longer period of time to complete. These tasks include problems that are not well defined and that demand research and investigation. They allow students to participate actively in the entire problem-solving process. During this process, the student is expected to make assumptions, decisions, predictions, and recommendations. In

addition, clearly communicated explanations and justifications are generally a major portion of the project.

Many extended projects require collaboration. Students may work in small groups or undertake a project that involves the whole class. In either situation, assessment by the teacher and both self and peer evaluation must be ongoing. The teacher's role is to monitor progress, provide resources, and to note demonstration of skills and attitudes of students throughout the project. Since most extended problems serve as both instructional and assessment opportunities, the teacher must also be aware of skills that students are struggling to learn.

The following example includes two possible sets of criteria for assessment:

The teachers would like to take the entire student body of your ***Middle School*** *on a field trip to Billings. Design a budget for the trip and prepare a presentation for the administration that details your budget. Keep in mind that the principal is very particular about details and will want a complete explanation of each expenditure.*

Example criteria:

- *Clearly Define the Problem*
- *Provide Documentation of Research*
- *Include a Detailed Budget*
- *Present a Clear Explanation of Each Expenditure Including Justification*

Example criteria:

Provide a detailed budget including a clear explanation of each item.

Include:
travel expenses,
food, and
admission costs.

Journals

Students are often asked to keep journals in courses that have traditionally required more writing. Now students are using journals in mathematics classrooms as well.

In addition to being a form of assessment, journals can serve as an instructional tool. They can be used to write explanations of mathematical problems and to record questions as they arise during the problem solving process. Journals can help students and teachers monitor progress and identify areas of particular difficulty. Students can use their journal to record and reflect on feelings and attitudes about learning mathematics. Perhaps most importantly, journals facilitate communication between teacher and students.

Journals can take many forms. Oftentimes, teachers prefer to structure journal writing so that a specific outcome can be evaluated. Other times, journals can be used as a way for students to pose questiongs or express feelings about mathematics and learning. Included below are two examples that can be used in a mathematics classroom.

Journal

Something that I understand well is...

Something that confuses me is...

Journal

Describe your favorite way(s) to learn mathematics.

Oral and Written Presentations

Presentations can be used in many situations within the mathematics classroom. Culminating activities designed to give students an opportunity to demonstrate what they have learned may involve a combination of formats. An oral presentation supported with written and visual information may be most appropriate for assessment of a long term project. Visual presentations, similar to those seen at science fairs, may be used to present solutions to problems or results of research. Written presentations are often appropriate for clarifying thinking processes, reporting of results, and justification of solutions, predictions, and recommendations.

The use of oral and written presentations should not be limited to culminating assessment activities. It requires a great deal of practice to learn to translate reasoning and problem-solving processes to written language. Assessment can be both integrated into instruction and an ongoing process when written and oral presentation of problem-solving processes are incorporated into daily classroom activities. Included here is an example writing assignment followed by a rubric detailing evaluation criteria.

Example Writing Assignment

Write a letter to the Austrians thanking them for the Austrian method of subtraction. Include a full explanation of both methods of subtraction (Austrian and borrowing) in both the Egyptian and Hindu-Arabic number systems.

Step One: In your group, do several subtraction problems using the Austrian method with Egyptian symbols. Take some time to think about how it can work.

Step Two: Verbalize an explanation of step one and then verbalize an explanation of regular subtraction with Egyptian symbols. (Each member of the group should verbalize his/her own thoughts, as well as listen to other group members.)

Do the following steps individually. You may, however, use peer assessment throughout the process.

Step Three: Write down an explanation of Austrian subtraction using Egyptian symbols.

Step Four: Write down an explanation of regular subtraction with Egyptian symbols.

Step Five: Repeat steps two, three, and four using Hindu-Arabic symbols.

Step Six: Organize your explanations. Think about how you can put them together so that they will make sense to someone else.

Step Seven: Compose a rough draft of the letter. Self-assess, edit, peer assess, and write a final dnraft.

Example Rubric

	Criteria	Self		Peer		Teacher	
		Ex	Inc	Ex	Inc	Ex	Inc
	Thank the Austrians						
	Thoughts are Well Organized						
	Ideas are Expressed Clearly						
	Written in Letter Form						
	Correct Grammar and Punctuation						
	Explain the Austrian System with Egyptian Symbols						
	Explain the Austrian System with Hindu-Arabic Symbols						
	Explain Borrowing Using Egyptian Symbols						
	Explain Borrowing Using Hindu-Arabic Symbols						

Observations

Even though it may not be common for teachers to assign grades based on in-class observations, this form of assessment has long been used for the improvement of instruction and learning. In fact, teacher observation can be a valid tool for assessment. Many essential skills for problem solving are not easily assessed in any other way.

Asking probing mathematical questions is a vital part of becoming a successful problem solver. Although journals and presentations may provide the teacher with some insight into the questions that students are asking during the problem-solving process, it is within the context of discussion that questions are formulated, presented, modified, and perhaps even answered. Only by observing or participating in the group process will the teacher be able to accurately judge student demonstration of the outcome.

Documentation of in-class observations is essential. Choosing particular outcomes (e.g., Asking Probing Mathematical Questions) to assess by oservation will make the process of documentation more manageable. In large classes, select a number of students or choose two or

three groups to observe during each class period, or use a checklist to record observations. When using a checklist, however, it may be beneficial to include a column for comments in order to record some examples of student behavior. Included below is an example form to organize the documentation of in-class observations.

Example Observation Record Sheet

Student Name: _____

Outcome Group Skills	Date	Observed Behavior
Model Listening Skills		
Employ Questioning Techniques		
Incorporate Constructive Criticism		
Utilize Peer Tutoring		
Utilize Peer Assessment		
Utilize Self-Assessment		

Questioning

One-on-one communication with a student is often an excellent way to gain valuable insights into students' thought processes and to identify misconceptions. Questioning techniques used by the teacher are often used to initiate this dialogue.

Questioning can be used in a very structured setting such as an interview. In an interview situation, the primary goal is to allow the student to demonstrate what he/she knows. The student must be comfortable and feel safe taking risks. Therefore, during an interview may not be the best time to include instruction. It is best to avoid ending the interview by showing the student the correct way to solve the problem. This may detract from the accomplishments that the student did demonstrate. Choose a separate setting to make corrections and clear up misconceptions.

Conference situations are somewhat less structured than interviews and may include one student or a small group of students. More opportunity for discussion and instruction is provided. The discussion format of a conference allows students to clarify their own thinking as well as demonstrate their skills. Conferences with a sampling of the class may provide adequate information to substantially improve curriculum and instruction.

The kinds of questions that teachers ask serve as a model for the questions that students ask themselves and their peers while learning and practicing the problem-solving process. Teachers should model probing mathematical questions. They should pose questions that require students to explain their thinking and reasoning process.

As is reflected in traditional assessment practices, monitoring and reporting student progress has been the major purpose of doing student assessments. However, first teach and then test is quickly becoming an outdated paradigm. Today, best practices suggest that assessment should be integrated with instruction in order to improve student learning. Authentic problem-solving situations are the best environment in which to assess problem-solving skills and the internalization of concepts. Student learning should be assessed at all levels and across the entire curriculum. Program improvement through improvement of instruction and learning is now the major focus of assessment practices.

The following are sample questions suggested in *Mathematics Assessment Myths, Models, Good Questions, and Practical Suggestions,* which is published by the NCTM.

- Would you please explain that in your own words?
- What assumptions do you have to make?
- How would you research that?
- Have you tried making a guess?
- What do you predict will happen?
- How would you explain this process to a younger child?

APPENDIX I:
PREPARING STUDENTS TO LEARN

Expectations of both teachers and students are rapidly changing in conjunction with the educational reforms that are being advocated by the *Standards*. Students are being required to work in significantly different environments than they may be accustomed. Problem-solving, communication, and reasoning skills are being taught using small groups and cooperative learning. Although this format closely simulates many real-world problem-solving situations students may not be accustomed to working in cooperative group situations.

Students need to practice group process and communication skills in order to use them effectively in cooperative learning situations. Before cooperative learning groups can be used productively, students must be comfortable in small group situations. The necessary communication skills must be directly addressed before students can be expected to use them effectively.

The following is an example of a unit designed to introduce group work. It is important to emphasize that these activities are for the purpose of exploring and practicing ways to learn. Therefore, the practice activities do not specifically target mathematics concepts.

Example Lesson Plans

OUTCOME: Use brainstorming and group discussion to establish rules, consequences, and rewards.

Activity One: Hats

1. Make construction paper hats using the following colors: white, red, black, yellow, green, and blue.

2. Using Handout One, ask each student to make an appropriate comment based on the color of hat that s/he chose to make.

   Examples:

   John is wearing a red hat. (White)

   I like John's red hat. (Red)

   John's hat will not stay on his head because it is too small. (Black)

   How does that suggestion relate to the facts given by Joanne? (Blue)

3. Homework: Identify at least one comment of each color that you actually hear someone say (or read). It may be in or out of school, on TV, or even in the newspaper. Please include the quote, who made the statement, the circumstance in which the comment was made, and the color(s) represented.

Example:

"The problem with his plan is that it requires approval from every employee."
Corporate Executive

This statement was made at a closed meeting of the accounting department of XXX Company.

The color represented is black because it points out a negative.

Activity Two: Establish Rules

1. Use think, pair, and share to come to a consensus on the rules that will be appropriate for the class.

Ask each student to think of five rules and write them down. Next, with a partner, discuss your list of ten rules and write down five that you both agree upon. (You may want to repeat the previous step in groups of four if you have a very large class.) Record each of the five final rules on separate sticky notes. For the last step, each group posts their five sticky notes on the board one group at a time. As each note is posted, it is observed whether or not it is similar to already posted rules. This allows for sorting into categories as the notes are posted.

Large group discussion may be most appropriate at this time. Remember, it is to your advantage as a teacher for all of your students to agree on the rules of your classroom.

2. Homework: Write a short paragraph justifying each rule. Include several examples of behavior that would not be allowed and at least one example of behavior that should be rewarded.

Activity Three: Establish Consequences and Rewards for Behavior

1. Discuss the Purpose of Brainstorming (to generate ideas)

This may be done in a large group format by asking for examples of situations that call for brainstorming and then generalizing the results.

Discuss the Rules of Brainstorming

(Write down all of the ideas generated, no analysis, no judgements, white, yellow, and green comments only.)

Include the reasons for these rules during the discussion. You may want to ask students to make written comments. Answer the question: Why are red and yellow comments not allowed?

2. Use the brainstorming process to generate a list of consequences (both positive and negative) for behaviors.

   This can be done in small groups or in a whole class situation. Remind the students that they should express all thoughts that come to mind. The sorting, categorizing, and discussion of ideas come later.

   Ask students to discuss when rewards are appropriate, who can reward students for behavior, and why it is important to acknowledge appropriate and outstanding behavior.

3. Assessment Opportunity: Submit a classroom discipline policy. Include justification for each component of the policy.

   You may want to prepare a rubric to include with the assignment.

HATS

While doing group problem-solving activities, many different "kinds of thinking" are required. We will use colored hats to explore six kinds of thinking that are often used in group situations. Each color of hat represents a way of thinking that is appropriate at some time in the process.

The ***Black Hat*** represents neutrality. "Just the facts, ma'am."

The ***Red Hat*** legitimizes emotions and feelings as an important part of thinking. It allows the thinker to say: "This is how I feel about the matter." It also allows a thinker to explore the feelings of others by asking for a red hat comment.

The ***White Hat*** is specifically concerned with negative assessment. It points out what is wrong, what does not fit with experience, why something will not work, and the risks, dangers, and faults in a design. Black hat thinking is not argumentive, it is an objective attempt to put the negative elements onto a map. Negative comments without justification (e.g. "That is a stupid idea.") are not black hat comments and are never appropriate.

Yellow Hat thinking is positive and constructive. It ranges from logical and practical at one end to dreams, visions and hopes at the other end.

Creative thinking belongs to the ***Green Hat.*** It generates new concepts and perceptions.

The control hat is blue. The ***Blue Hat*** controls the thinking process, calls for the use of other hats, and keeps the discussion moving in an organized way.

Handout One

OUTCOME: Analyze, Interpret, and Evaluate the Group Process

Activity One: Define the Components of the Group Process

1. Lead a discussion of past experience working in groups.

   Example questions: What groups are you a member of? When do you work in groups? Does everyone always do their share? Was the group ever confused about what it was supposed to accomplish? Have you ever had to work with someone that you did not like? Did the group ever lose track of time and not finish a task? Did anyone ever put someone else down? Have you had the opportunity to meet and work with someone that you really like?

2. Ask the questions: Why do people work in groups? What are the purposes of working in groups in the classroom?

   Purposes should include that people are more productive in groups and that working together has social value (i.e., it's more fun, it makes people feel good).

3. What are the rules/guidelines for working in groups? Are they different for different groups?

   Students should establish that integrity and cooperation are keys to successful group processes and that everyone must be responsible for the learning.

Activity Two: Practice Using the Group Process

1. Read the poem Sego Lily by Idabell Cramer.

   Why does the Sego Lily make the poet cry?

   Agree on a best answer and be certain that everyone can use the poem to justify that answer.

   You may want to ask each group to report to the class. Also, ask each student to reflect upon the process. Did everyone follow the guidelines? What made the group effort successful? What could have been improved?

SEGO LILY

Sego Lily, Sego Lily,
With your petals white as snow
Nodding to me as the breezes
Gently sway you to and fro.

Sego Lily, Sego Lily,
Now my tears will softly flow
As I wander in my memories
Back to days of long ago.

To a cabin in a clearing
It was there that I first knew
How the perfume of your presence
Is enhanced by morning dew.

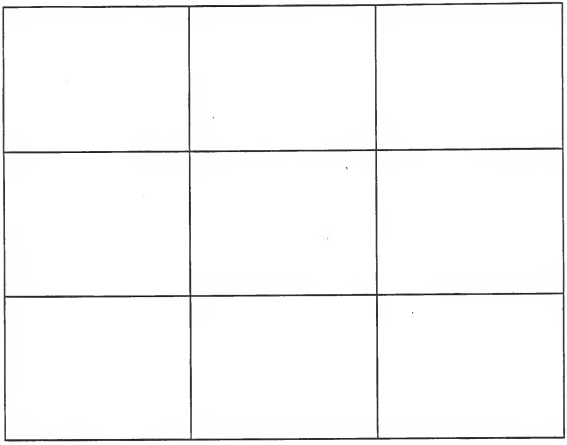
It was there the man I'd marry
Picked your blossom Oh so fair.
Sego Lily, Sego Lily,
There he placed it in my hair.

Many years we've been together
Watching seasons come and go
And each spring the Sego Lily
Says that God still wants it so.

by Idabel Cramer

2. Use Handout Two

   Ask students to report to the large group and to reflect on the processes used during problem solving.



How many boxes?

Explain your process.

APPENDIX II: RESOURCES

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